3/2/17 "I make the most of all that comes and the least of all that goes."-Sara Teasdale

HW: "Average Rate of Change" homework section
Test 2 on Tuesday 3/14

AIM: How do we find Average Rates of Change?

Warm Up:

Determine the equation of the parabola whose focus is (0,8) and whose directrix is the horizontal line y = 2?

Distance to Directrix = Distance to Focus

$$(y-2)^2 = \left(\sqrt{(x-0)^2+(y-8)^2}\right)^2$$

$$\cancel{y^2} - 4y + 4 = x^2 + \cancel{y^2} - 16y + 64$$

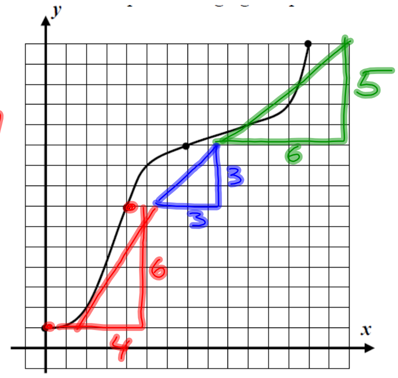
$$-\cancel{y^2} + 16y - 4 \qquad\qquad -\cancel{y^2} + 16y - 4$$

$$\frac{12y}{12} = \frac{x^2}{12} + \frac{60}{12}$$

$$y = \frac{1}{12}x^2 + 5$$

***Exercise* #1:** The function $f(x)$ is shown graphed to the right.

(a) Evaluate each of the following based on the graph:



(i) $f(0)$ $= \boxed{1}$ (ii) $f(4)$ $\boxed{7}$ (iii) $f(7)$ $\boxed{10}$ (iv) $f(13)$ $\boxed{15}$

(b) Find the change in the function, Δf over each of the following domain intervals. Find this both by subtraction and show this on the graph.

(i) $0 \le x \le 4$

$f(4) = 7$
$f(0) = 1$
$7 - 1 = 6$
$\boxed{\Delta f = 6}$

(ii) $4 \le x \le 7$

$f(7) = 10$
$f(4) = 7$ $10 - 7 = 3$
$\boxed{\Delta f = 3}$

(iii) $7 \le x \le 13$

$f(13) = 15$
$f(7) = 10$ $15 - 10 = 5$
$\boxed{\Delta f = 5}$

(c) Why can't you simply compare the changes in f from part (b) to determine over which interval the function changing the fastest?

The intervals are not the same.

(d) Calculate the **average rate of change** for the function over each of the intervals and determine which interval has the greatest rate.

AROC $\frac{\Delta f}{\Delta x}$

(i) $0 \le x \le 4$

$\frac{\Delta f}{\Delta x} = \frac{6}{4} = \frac{3}{2}$

$\frac{\Delta f}{\Delta x} = 1.5$

Greatest Rate

(ii) $4 \le x \le 7$

$\frac{\Delta f}{\Delta x} = \frac{3}{3} = 1$

$\frac{\Delta f}{\Delta x} = 1$

(iii) $7 \le x \le 13$

$\frac{\Delta f}{\Delta x} = \frac{5}{6}$

$\frac{\Delta f}{\Delta x} = .833$

(e) Using a straightedge, draw in the lines whose slopes you found in part (d) by connecting the points shown on the graph. The average rate of change gives a measurement of what property of the line?

Average Rate of change tells us the SLOPE

Average Rate of Change

For a function over the domain interval $a \le x \le b$, the function's **average rate of change** is calculated by:

$$\frac{\Delta f}{\Delta x} = \frac{\text{change in the output}}{\text{change in the input}} = \frac{f(b)-f(a)}{b-a} = \frac{y_2-y_1}{x_2-x_1}$$

line parabola

Exercise #2: Consider the two functions $f(x)=5x+7$ and $g(x)=2x^2+1$.

(a) Calculate the average rate of change for both functions over the following intervals. Do your work carefully and show the calculations that lead to your answers.

(i) $-2 \le x \le 3$

$$\frac{f(3)-f(-2)}{3--2} = \frac{22-(-3)}{5} = \frac{25}{5} = 5$$

$$\frac{g(3)-g(-2)}{3--2} = \frac{19-9}{5} = \frac{10}{5} = 2$$

(ii) $1 \le x \le 5$

$$\frac{f(5)-f(1)}{5-1} = \frac{32-12}{4} = \frac{20}{4} = 5$$

$$\frac{g(5)-g(1)}{5-1} = \frac{51-3}{4} = \frac{48}{4} = 12$$

(b) The average rate of change for *f* was the same for both (i) and (ii) but was not the same for *g*. Why is that?

f(x) is a line and the slope stays the same.

g(x) is a parabola which has changing slope.

Exercise #3: The table below represents a linear function. Fill in the missing entries.

x	1	5	11	19	45
y	-5	1	10	22	61

4

6

AROC is the same

$$\frac{\Delta f}{\Delta x} = \frac{1-(-5)}{5-1} = \frac{6}{4} = \frac{3}{2}$$

$$\frac{y-1}{11-5} = \frac{3}{2}$$

6

$2y-2=18$

$2y=20$

$y=10$

12

$$\frac{22-10}{x-11} = \frac{3}{2}$$

$24=3x-33$

$57=3x$

$19=x$

$$\frac{y-22}{45-19} = \frac{3}{2}$$

26

$2y-44=78$

$2y=122$

$y=61$